

## Chapter 6. Polygons, Circles, Stars and Stuff



“Now it’s time for the magic!”

“Magic?” asked Morf. “What do you mean, magic? You’ve never talked about Logo magic before.”

“We’ve talked about shapes, and how you can put two or more together to make a picture. You’ve talked about writing procedures and adding variables and things to those procedures.

“But this is all pretty common stuff. Now it’s time to discover some of the magic...not just Logo magic but math magic.”

---

### Playing With Polygons

Let’s start with that hexagon you were working with in the last chapter.

```
TO HEXAGON :N  
REPEAT 6 [TRI :N RIGHT 60]  
END
```

Remember the TRI :N procedure that was in that HEXAGON procedure? Let’s give the :N variable a value of 100. So now it becomes...

```
TO TRI  
REPEAT 3 [FD 100 RIGHT 120]  
END
```

Take a look at your other shape procedures.

Square: REPEAT 4 [FD 100 RIGHT 90]

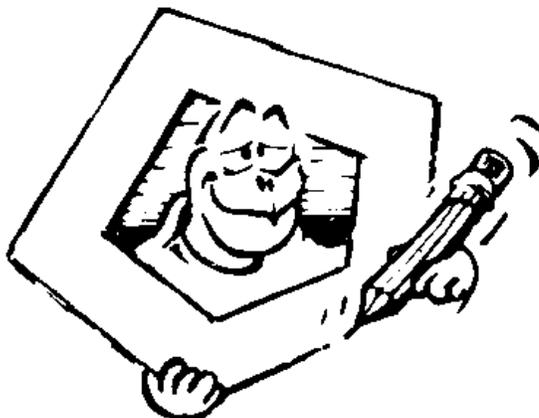
Rectangle: REPEAT 2 [FD 100 RT 90 FD 200 RT 90]

---

## Pentagon Power

You've worked with three, four, and six-sided shapes. What about a five-sided shape? What do you call that?

That's a pentagon, just like that big building near Washington, D.C.



Pentagon: REPEAT 5 [FD 100 RIGHT \_\_\_\_]

But how do you know how far the turtle should turn? You can try a few numbers to see what works. But rather than waste a lot of time, let's see if there's something you can learn from the other shape procedures.

Look at the triangle. 3 repeats times 120 turns = \_\_\_\_\_

How about the square? 4 repeats times 90 turns = \_\_\_\_\_

The hexagon? 6 repeats times 60 turns = \_\_\_\_\_

The rectangle? 2 repeats times  $90 + 90 =$  \_\_\_\_\_

What goes in the blanks? Bet it's 360 turns. (There's that number again.) So, if you want to write a procedure for a pentagon, how about this?

**REPEAT 5 [FD 100 RIGHT 360/5]**

Try it... What happens?

---

## Any Number of Sides

If you can do that for a 5-sided shape, bet you can do that for any number of sides. Try that, too...

**REPEAT 3 [FD 100 RIGHT 360/3]**

**REPEAT 4 [FD 100 RIGHT 360/4]**

**REPEAT 6 [FD 100 RIGHT 360/6]**

**REPEAT 9 [FD 100 RIGHT 360/9]**

**REPEAT 15 [FD 100 RIGHT 360/15]**

Now make up some shape of your own.

**REPEAT \_\_\_\_ [FD 100 RIGHT 360/\_\_\_\_]**

What happens when the number of repeats get pretty big? What does the shape look like?

## Polygons, Circles, Stars and Stuff

One of the first things you notice is that the shapes begin to go off the screen. When this happens, make the number of FORWARD steps smaller.

**REPEAT 90 [FORWARD 5 RIGHT 360/90]**

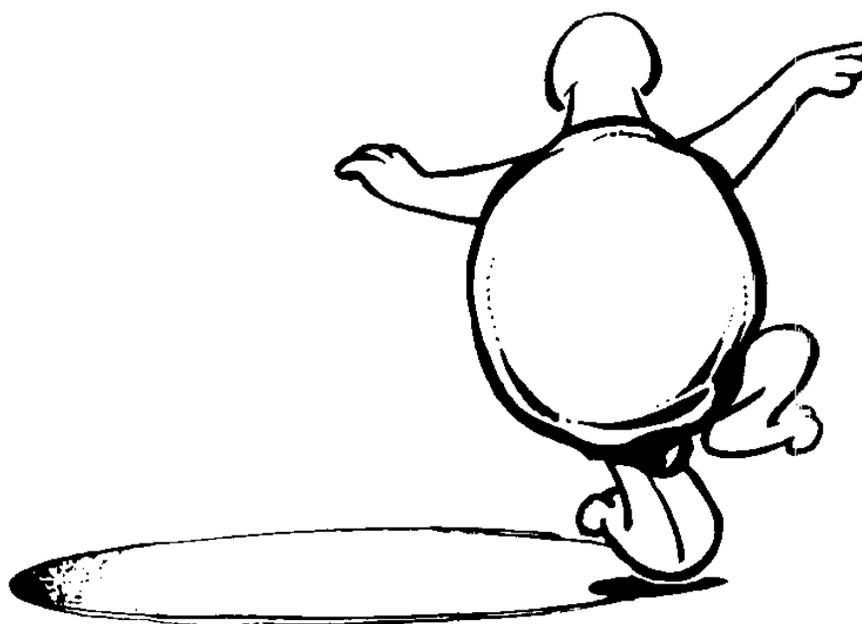
**REPEAT 180 [FORWARD 2 RIGHT 360/180]**

The higher the number of repeats, the more the shape begins to look like a circle.

---

## Playing With Circles

If that's true, then what would the procedure for a circle be?



**REPEAT 360 [FORWARD 2 RIGHT 360/360]**

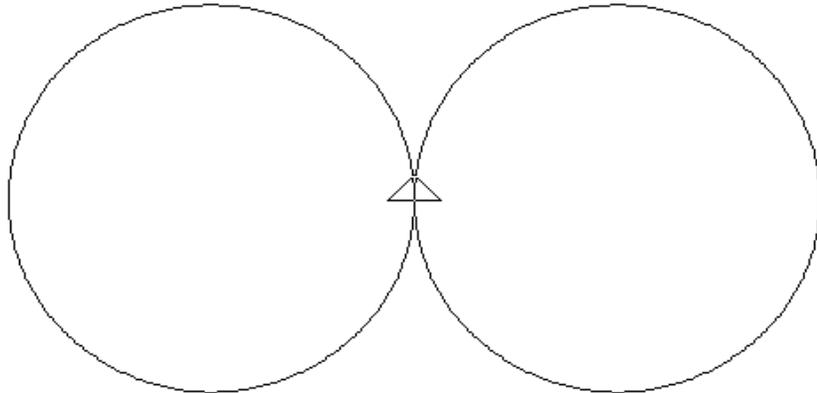
That's the same as...

**REPEAT 360 [FORWARD 2 RIGHT 1]**

Of course, circles don't have to go to the right. Why not try a circle to the left?

**REPEAT 360 [FD 2 LEFT 1]**

**REPEAT 360 [FD 2 RIGHT 1]**



Gee, sort of looks like cross-eyes, doesn't it?

---

## **A Circle Challenge**

How would you put two smaller circles inside these big ones? Solving this challenge is like solving any problem.

1. Start with what you know.

The turtle just drew two large circles by going FORWARD 2 steps, RIGHT...and LEFT...1 turtle turn, 360 times.

2. Now...what do you need to know?

You need to know how to change the procedure so the circles will be smaller.

There are three things you can change.

You can change the number of repeats.

You can change the number of turtle steps.

You can change the number of turtle turns.

## Polygons, Circles, Stars and Stuff

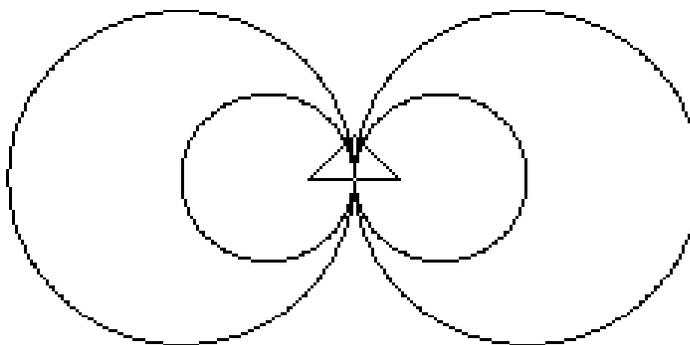
But...you have already learned that to draw a circle, you tell the turtle to...

**REPEAT 360 [FORWARD :N RIGHT or LEFT 1]**

So...to draw a smaller circle, seems like you can make the number of steps smaller. So try this...

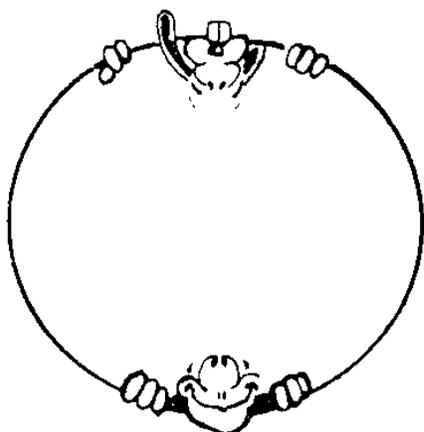
**REPEAT 360 [FORWARD 1 RIGHT 1]**

Looks OK, doesn't it. Draw a circle to the left.



Now that really looks like cross-eyes. Why not try some other drawings using circles and the other shapes you've been using?

Draw some circle shapes here or in your journal. And then draw them on the screen.



What else do you know about drawing a circle?

You know that the number of repeats times the number of turtle turns equals 360. You used 360 times 1. What happens if you change that to 180 times 2? Would that make a smaller circle or a larger one?

Go ahead. Try it out.

Earlier, you started with the familiar shapes and increased the number of REPEATS and the number of Turns. Now let's turn this around and go the other way...just for the fun of it.

REPEAT 360 [FORWARD 2 RIGHT 1]

REPEAT 180 [FORWARD 2 RIGHT 2]

REPEAT 120 [FORWARD 2 RIGHT 3]

REPEAT \_\_\_ [FORWARD 2 RIGHT \_\_\_]

Pretty soon, it stops looking like a circle.

---

## Defining a Circle

Just what is a circle...really? You made up some rules about other shapes. So what is the rule for circles?

If you ask a math teacher, you'll find that a circle is a closed line where every point on the line is the same distance from a fixed point. On the Logo screen, you can use HOME as the fixed point. Then a circle is a closed line where every point on that line is the same distance from HOME.

Make sense? Sure! Now...can you write a procedure to draw a circle like this?

Let's make HOME our fixed point...the center of our circle. Next you have to draw a line so that every point on that line is the same distance from HOME. How can you do that?

You'll have to pick the pen up, go out to where you want your circle to be, draw a point, come back, turn a little bit, go out the same distance, draw another point, come back, turn a little...and so on.

Before you start drawing the line, let's first write a procedure to draw a point.

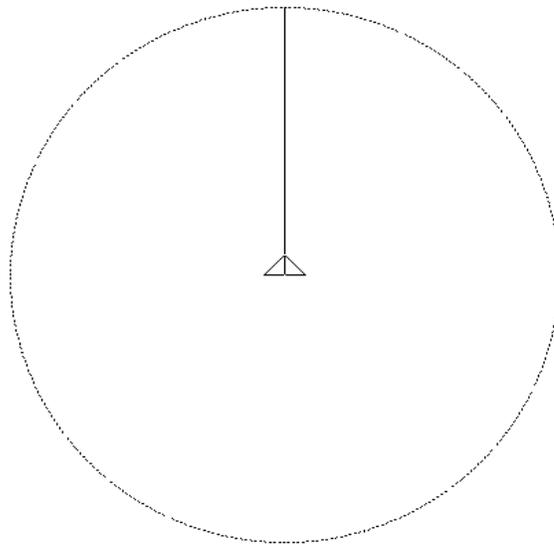
```
TO POINT  
PD RT 90 FD 1  
BK 1 LT 90 PU  
END
```

See what that does?

Now let's put this procedure to good use.

```
TO CIRCLE  
HOME CS  
REPEAT 360 [FD 200 POINT BK 200 RT 1]  
END
```

The procedure starts from HOME with a clear screen. Now, rather than go FD 1 and RT 1, the turtle goes FD 200, draws a point, goes back HOME, and turns right 1 turtle turn. You draw a line where every point on that line is the same distance...200...from the center.



What do you call that distance from the center of the circle...the fixed point...to the edge?

That's the radius of the circle. What do you call that edge...the line around the fixed point?

Right...that's the circumference of the circle.

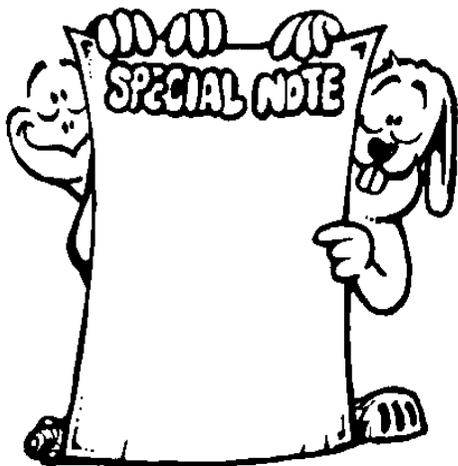
Another one...what do they call the distance from one side of the circle to the other through the center of the circle?

That's the diameter of the circle.

One more...what's the relationship between the diameter of the circle and the circumference?

Hmmm...that's a tough one isn't it.

The diameter times Pi equals the circumference. But what's Pi?



---

**SPECIAL NOTE:** Pi is the 16th letter in the Greek alphabet. It is also one of those strange mathematical numbers that will keep your computer busy for hours. They call it a “transcendental” number because it has no set value.

There is a procedure that comes with MSW Logo that calculates Pi to as many decimal places as you want. Here's Pi calculated to 50 places.

3.1415926535897932384626433832795  
02884197169399375105

Load the procedure and type PICALC and the number of decimal places to which you want Pi calculated. For example, PICALC 50.

---

## The Rule of 360

Now...remember what you learned about that number 360? There's another rule in there somewhere.

Seems that when you add up the angles you need to draw any closed shape that is made from straight lines, the answer is 360.

Another interesting thing is that when the turtle travels through 360 degrees, she always ends up where she started.

Try it. Add the same number to both blanks in this procedure.

```
REPEAT _____ [FD :SIDE RT 360/_____]
```

**Take a look at these commands again. How would you write one procedure that would draw all of them?**

**Triangle:**

**REPEAT 3 [FORWARD :SIDE RIGHT 360 / 3]**

**Square:**

**REPEAT 4 [FORWARD :SIDE RIGHT 360 / 4]**

**Pentagon:**

**REPEAT 5 [FORWARD :SIDE RIGHT 360 / 5]**

**Hexagon:**

**REPEAT 6 [FORWARD :SIDE RIGHT 360 / 6]**

**Octagon:**

**REPEAT 8 [FORWARD :SIDE RIGHT 360 / 8]**

**In these procedures you use the variable :SIDE for the length of one side of the shape.**

**How about this one?**

**TO SHAPE :SIDE**

**REPEAT \_\_\_\_\_ [FORWARD :SIDE RIGHT 360 / \_\_\_\_\_]**

**END**

**When you fill in the blanks like you did above, you can draw just about any shape.**

**How about this...**

**TO POLYGON :SIDE :REPEATS**

**REPEAT :REPEATS [FD :SIDE RT 360/ :REPEATS]**

**END**

Once you've played with this procedure a bit, write one to draw all of the shapes up to a circle.

Sure you can do it. Give it a try.

```
TO POLYGONS :SIDE :REPEATS
IF :REPEATS = 360 [STOP]
REPEAT :REPEATS [FD :SIDE RT 360 / :REPEATS]
POLYGONS :SIDE :REPEATS + 1
END
```

---

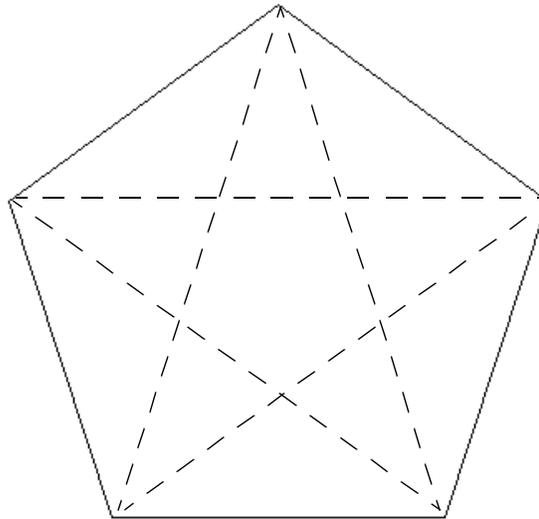
### Rabbit Trail 17. Star Gazing



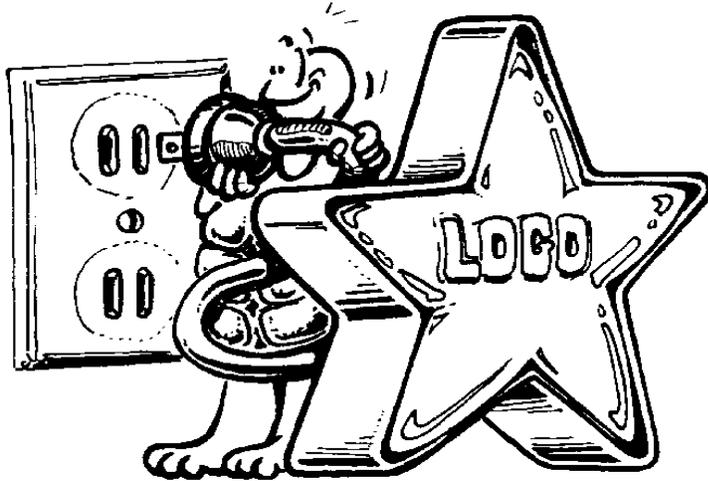
Draw a big pentagon on the screen. Remember how to do that?

```
REPEAT 5 [FD 100 RT 360 / 5]
```

Now print the screen. Take a pencil and a ruler and draw a diagonal line to each corner. In this way, every corner is connected to every other corner by a line.

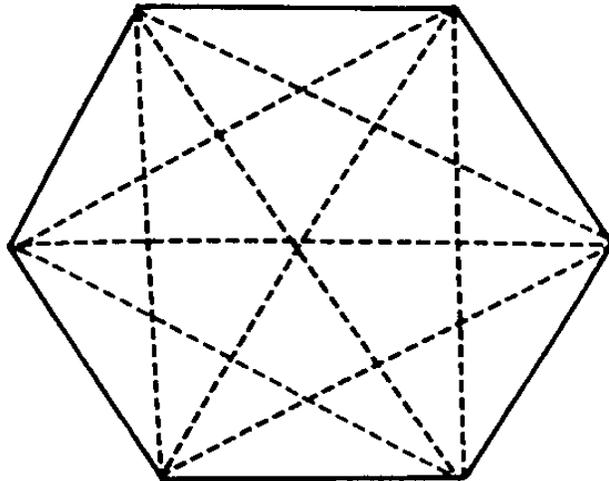


What do you see inside the pentagon? I see a big star, how about you?



Now draw a big hexagon on the screen.

**REPEAT 6 [FD 100 RT 360 / 6]**

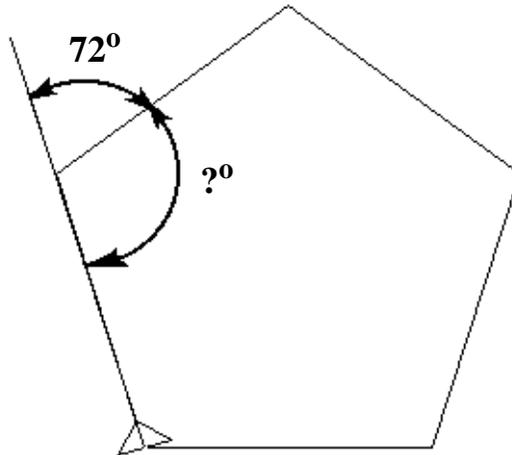


**Draw diagonal lines to connect all the corners and what do you see? There's a Star of David, right? That's a six-pointed star.**

**Try this with an octagon. Now that's a lot of lines to draw.**

**How would you draw a star on the screen? Do the drawings you did on paper help you figure it out?**

**Take a look at the pentagon. To draw the shape, the turtle goes `FD 100 RT 360 / 5` -- that's `RT 72`. The angle outside the shape is 72. What's the angle inside the shape?**



**Let's find out. Clear the screen and draw another pentagon like the one above. Make sure you can see the turtle. Then type...**

**FD 100**

**The turtle has now moved up the first side of the pentagon, right? What's the command to make the turtle turn around and head back to HOME?**

**That's not BACK 100.**

**The turtle doesn't turn around using the BACK command. The command to use is RT or LT 180, isn't it...half a circle...like the two angles shown in the drawing above. Go ahead...try it.**

**RT 180 FD 100**

**Now where's the turtle?**

Type HOME and see what happens.

All the turtle does is turn around. She's back home. So let's try something else. Turn LT 18 and head up to left side of the Pentagon.

FD 100 RT 72

Now the turtle is heading up the second side of the pentagon.

Now the big question? How far do you have to turn to send the turtle HOME?

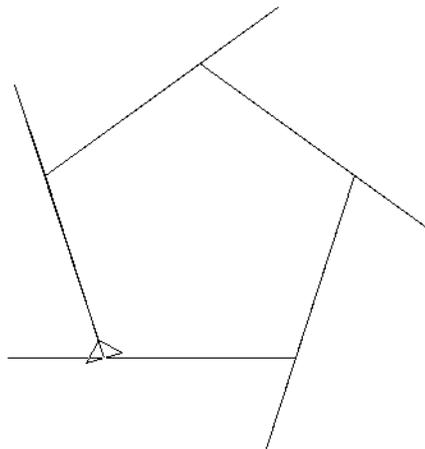
That's not that hard, really. You turned 180 the first time, right? What's  $180 - 72$ ? I get 108. What do you get? Try it...

RT 108 FD 100 HOME

There you are...Ernestine is right where she was before.

Believe it or not, you're actually getting somewhere. Don't believe me? Well, try this...

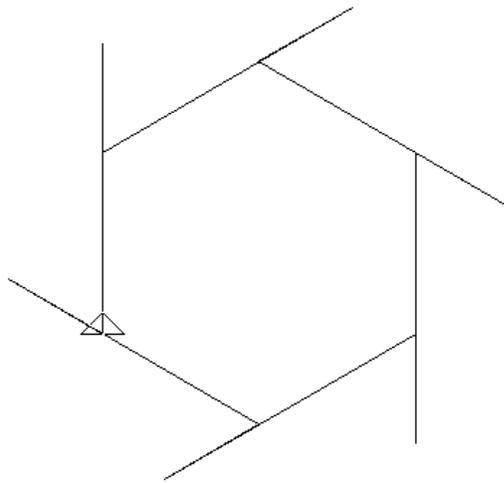
LT 18 REPEAT 5 [FD 150 BK 50 RT 72]



**Wow...that's a pretty weird pentagon! But it shows you what you've been doing. To draw a pentagon, Ernestine goes FD 100, turns RT 72. Now you can see the angles much more clearly.**

**Try the same thing with the hexagon.**

**REPEAT 6 [FD 150 BK 50 RT 60]**



**When drawing a hexagon, Ernestine goes FD 150, BACK 50, and turns RT 60 on the outside. That leaves an angle of \_\_\_\_\_ on the inside?**

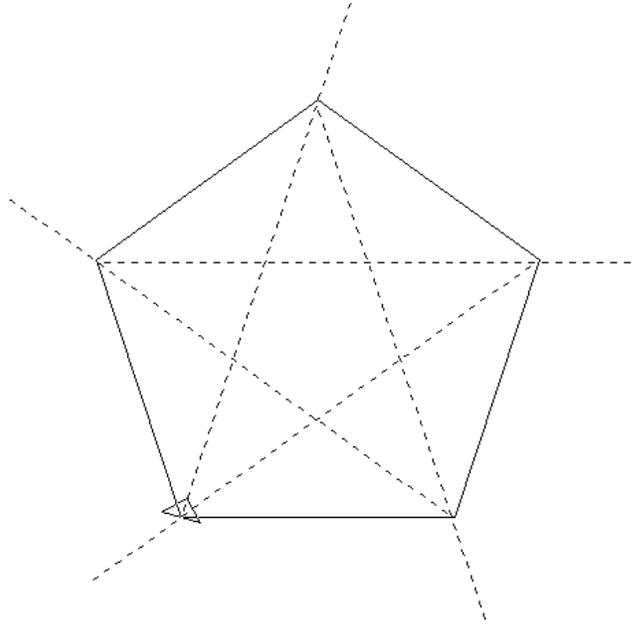
**If you said 120, let's make you a big Gold Star!**

**Look at your drawing of the pentagon with the diagonals. At each corner, how many angles do you see? I see three, how about you?**

**Are all the angles the same? OK...so if that inside angle is 108, what are each of the three angles? That's 108 divided by 3 or 36, right?**

**Now... take your pencil and ruler and draw along any one of the diagonal lines to make it go out beyond the**

pentagon...like Ernestine did went she went FD 150 BK 50. Here you have the large angle outside the star and the small angle inside. You already know the small angle is 36. So what's the big angle?



That's  $180 - 36$  or...144.

Now you're *really* getting somewhere! So what angle do you need to draw a star?

REPEAT 5 [FD 160 RT \_\_\_\_\_]

Now add this one...

LT 36 REPEAT 5 [FD 100 RT 72]

How about that! There's the star inside the pentagon...just like the picture you drew on paper. But how'd Ernestine know the distance was 160? Well, that's another lesson.

Why not see what you can do with the Star of David now? If you look closely, you'll see there are two triangles in the star.

## Polygons, Circles, Stars and Stuff

Here's a hint. If the side of the hexagon is 100, try 172 for the side of the triangle.

```
TO PENTAGON
REPEAT 5 [FD 100 RT 72]
END
```

```
TO STAR
RT 36 REPEAT 5 [FD 160 RT 72]
END
```

```
TO HEXAGON
REPEAT 6 [FD 100 RT 60]
END
```

```
TO STAR.OF.DAVID
RT 30 TRI
RT 90 FD 100
LT 150 TRI
END
```

```
TO TRI
REPEAT 3 [FD 172 RT 120]
END
```

---

### Something Else to Think About

How would you draw a seven-pointed star? A seven-sided polygon is easy...

```
REPEAT 7 [FD :SIDE RT 360 / 7]
```

Now turn that into a star. Then see what you can do with 8, 9, 10, 11 points...and other star shapes.

---

**REPEATS \* :N** Remember our “any polygon” command?

**REPEAT :REPEATS [FD :SIDE RT 360/ :REPEATS]**

To draw a pentagon, the command would be...

**REPEAT 5 [FD :SIDE RT 360/5]**

What happens when you type...

**REPEAT 5 [FD 100 RT 360/5 \* 2]**

That’s repeat the following five times: Forward 100, turn right 360 divided by 5, or 72, times 2 or 144. It’s the same as...

**REPEAT 5 [FD 100 RT 144]**

Does the shape this draws look familiar? It should. There’s the star again. Now let’s try that same thing with the hexagon.

**REPEAT 6 [FD 100 RT 60]**

Let’s change that to...

**REPEAT 6 [FD 100 RT 60 \* 2]**

When you tried this with the pentagon, you went from a five-sided shape to a fine-pointed shape. When you did the same thing with the hexagon, did you go from a six-sided shape to a six-pointed shape?

What happened? Why?

## Polygons, Circles, Stars and Stuff

How about an octagon?

**REPEAT 8 [FD 100 RT 45]**

Let's change that to...

**REPEAT 8 [FD 100 RT 45 \* 2]**

What would happen if you changed the \* 2 to \* 3?

Do some experimenting with different numbers to see what happens. Among other things, you'll find you have to change the Rule of 360 a little bit.

Before we said...

“Another interesting thing is that when the turtle travels through 360 degrees, she always ends up where she started.”

Now we have to change that to read...

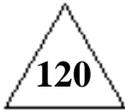
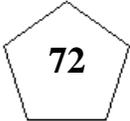
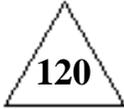
“Another interesting thing is that when the turtle travels through 360 degrees...or any multiple of 360...she always ends up where she started.”

## Rule of 360 Worksheet

Fill in the correct numbers in the following chart and draw a picture of what happens on the screen. A few have been filled in to get you started.

The numbers at the top of each column in the table represent the number of repeats. The numbers down the left column represent the number of turtle turns.

For the first row across the top of the table, the number at the top of each column equals the number of sides for the shape.

Number of Turtle Turns (TT)	Number of Repeats						
	3	4	5	6	8	10	12
TT * 1							
TT * 2							
TT * 3	360						
TT * 4		360					
TT * 5			360				

## Circles, Curves, Hoops, and Things

By now, you should have a pretty good idea of how to work with shapes...the square, rectangle, triangle, circle, and such.

Now it's time to start making Logo jump through some hoops! In the rest of this book, you'll find there's a lot more to turtle graphics than just drawing shapes.



You already know that you can draw a circle using a procedure like this one.

```
REPEAT 360 [FORWARD 1 RIGHT 1]
```

How would you draw a curve? How would you draw just part of a circle?

If there's 360 degrees in a circle, part of a circle would be something less than that. So let's try...

```
REPEAT 90 [FD 1 RT 1]
```

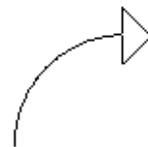
Hmmm...

Think about this. You already know that RT or LT 180 makes the turtle turn around and go the opposite direction. So what would happen "did a 180" here?

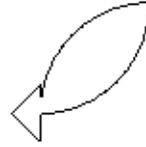
```
REPEAT 90 [FD 1 RT 1]
```

```
RT 90
```

```
REPEAT 90 [FD 1 RT 1]
```



I don't know about  
you but that sorta  
looks like a leaf to me.

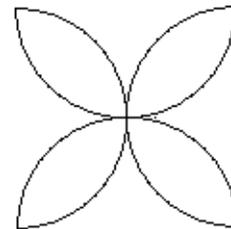


What would happen if you just continued around the  
circle? Here's the leaf...

```
TO LEAF  
REPEAT 90 [FD 1 RT 1]  
RT 90  
REPEAT 90 [FD 1 RT 1]  
END
```

Now try this...

```
TO LEAVES  
REPEAT 4 [LEAF]  
END
```

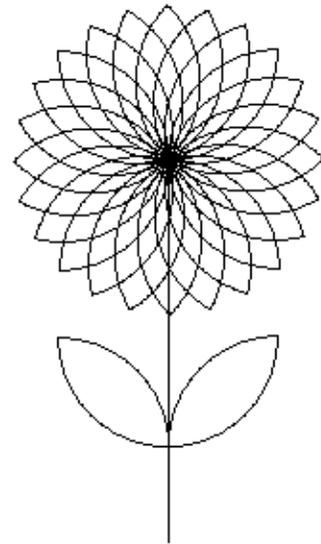


You know...with a little bit of work, that could be all  
sorts of things. Why not see what you can do with this  
shape before you move along? Maybe it could be a  
butterfly? Or maybe a Indian headdress?



**How about a chrysanthemum?**

**How about making a  
whole flowerbed  
using different sizes  
of curves...different  
sizes of leaves?**



**What other types of flowers can you draw?**

---